INTRODUCTION

Performance-based earthquake engineering (PBEE) has received increasing attention among structural engineering researchers and practitioners (Ghobarah 2001) in order to predict the performance of a structure subjected to earthquakes in a probabilistic manner and to design accordingly to achieve selected performance objectives (Ghobarah 2001, Porter et al. 2007). In the conventional PBEE framework, four parameters are used to compute annual loss rate of a structure due to earthquakes. The first parameter is intensity measure (IM), which quantifies the intensity of an earthquake ground motion, such as the peak ground acceleration and the spectral acceleration at the first mode period. The second parameter is engineering demand parameter (EDP), which represents the structural response to the earthquake, such as the peak story drift ratio or the absolute floor acceleration. The third is damage measure (DM), which describes the discrete physical damage state of the structure, such as cracking and spalling. The last is decision variable (DV), which relates to the actual loss, such as casualties, downtime, and monetary loss (Singhal & Kiremidjian, 1995). A fragility function represents the probability distributions of DM for various levels of IM. In other words, it provides the conditional probability of a structure being in a specific damage state, such as slight, moderate, or severe damage given a certain intensity of an earthquake. This relationship between IM and DM involves many uncertainties due to inherent randomness in ground motion and structural properties as well as modeling uncertainties and measurement errors.

Recent developments in analytical models of structures, numerical simulations, and various instrumentations, and numerous efforts to conduct post-earthquake assessments and to collect field data have provided considerable amount of data on structural damages that can be correlated to observed or inferred ground motion intensity. Among many ways these data can be organized and analyzed, this paper addresses issues related to the development of analytical and empirical fragility functions. The most common forms in which data are collected following an earthquake are identified, and the techniques to manipulate these data to a format that is useful for fragility function development are discussed. Three new approaches for fitting fragility functions to the data are introduced that enable the treatment of uncertainty in either or both of ground motion intensity and structural damage. The methods are applied to a set of numerically simulated data for a four-story steel moment-resisting frame and a set of field observations collected after 2010 Haiti earthquake. The results demonstrate that these methods can develop continuous representation of fragility functions without specifying their functional forms and treat sparse data sets more efficiently than conventional data binning methods.
fitting by assigning weights based on the distance from the target point.

Noh & Kiremidjian (2011) and Noh et al. (2011) introduced a kernel smoothing based framework for developing fragility functions using features from structural health monitoring, and the method is further extended and applied in this paper. The conventional methods to estimate fragility functions from analytical and empirical data include a data binning and a distribution fitting. The data binning methods divides IM values into several bins of a width, \( \Delta \text{im} \), and then computes the probability distribution of damage at each bin (Porter et al. 2007). One of the main issues with data binning for fitting a fragility curve is that the resulting curve is very sensitive to the subjective choice of binning. It also produces discontinuous fragility functions. On the other hand, the distribution fitting method fits a fixed cumulative probability distribution function (CDF) to the data, such as a lognormal distribution, using various methods such as maximum likelihood estimation and Bayesian analysis. This fitting can be done after applying the data binning method. The distribution fitting method provides a smooth fragility function, but specifying a functional form can restrict the choice of relationship and may not reflect the true structure of data. Kernel smoothing methods allow us to estimate continuous fragility functions without limiting them to specific functional forms. In addition, they can reduce the bias due to discretization (e.g., data binning) and provide more detailed information about the probabilistic relationship between DM and IM. Using the kernel smoothing methods is particularly beneficial when we have non-homogeneous sparse data. They provide statistically more stable results when the data are sparse because it utilizes all the observations to estimate the fragility functions unlike the data binning method which limits the data by the value of the IM.

This paper is organized as follows. Section 2 introduces three approaches for developing fragility functions from these data based on kernel smoothing methods. In section 3, these techniques are applied to a set of data collected from analytical model of a four-story steel moment-resisting frame and a set of field observations collected after 2010 Haiti earthquake, and the results are presented and compared with other conventional methods. Finally, section 4 provides a summary and conclusions.

2 GAUSSIAN KERNEL SMOOTHING (GKS) METHODS

A kernel is a weighting function assigned to each noisy observation, and the weight is inversely related to the distance between the observed value and the value that we want to estimate (Wand, 1995). In other words, if we are interested in computing the fragility function for damage measure \( dm^* \) at ground motion intensity \( im^* \), then the weight for a data point \((dm_i, im_i)\) is a function of the distances between \((dm^*, im^*)\) and \((dm_i, im_i)\), and a data point with larger distance is assigned a smaller weight than other data points with shorter distances. This implies that the data points farther away from the point of interest contribute less to the computation of the fragility function than those that are closer. The GKS uses the Gaussian probability density function with mean \((dm^*, im^*)\) as a weighting function. The variance of the Gaussian function represents the uncertainty of the data, and, if \( dm \) is continuous, using the multi-variate Gaussian function we can consider the uncertainties of both \( dm \) and \( im \) and their correlation. This GKS approach is particularly beneficial when the data are sparse because it efficiently utilizes the entire data set to compute the fragility function, instead of confining the data into different bins. In addition, the GKS avoids the discretization error unlike the data binning method. For example, a small measurement error may assign a sample to a different bin in data binning method, which may result in a large difference in regression. Also, all the data within a bin is considered equivalently no matter what their actual IM values are. On the other hand, the GKS uses a continuous kernel to assign a weight to each sample, so the effect of a small measurement error on the regression result is mitigated and their actual IM values contribute to computation of their weights.

Similarly, a kernel can represent the uncertainty in the observed value. We can estimate a probability density function (PDF) using the kernel as

\[
\text{Prob}(X = x) = f_X(x) = \frac{1}{n} \sum_{i=1}^{n} K\left(\frac{x - x_i}{h}\right)
\]

where \( x_i \)s are \( n \) realizations of the random variable \( X \), \( K \) is a kernel, and \( h \) is a smoothing parameter or the bandwidth of the kernel \( K \). For Gaussian kernel, \( K \) is a standard Gaussian probability density function, and \( h \) corresponds to the standard deviation.

\[
f_X(x)
\]

Figure 1. Example of kernel density estimation

Figure 1 shows an illustrative example of the Gaussian kernel density estimation based on the five observations, \( x_1, x_2, \ldots, x_5 \). The green dots indicate the observed values. The red dotted line indicates the kernel for each observation representing the un-
uncertainty of the observation and/or the contribution of the observation for computing the PDF of X at each value of x. Therefore, the bandwidth of the kernel (i.e., the standard deviation of the Gaussian kernel) can be selected on the basis of the measurement error, if available, or the engineering judgment. If the observations are iid samples of a Gaussian distribution, we can use Silverman’s optimum bandwidth \( h \) for the Gaussian kernel (Silverman, 1986). It is given as

\[
h = 1.06 \hat{\sigma} n^{-\frac{1}{5}} \tag{2}
\]

where \( \hat{\sigma} \) is the sample standard deviation, and \( n \) is the number of samples. Finally, the blue dash line is the estimated PDF using the kernels. Note that the resulting PDF does not have an explicit parametric form. Instead, it is represented as a sum of kernels whose number equals the size of the observations. For this reason, the kernel methods are often used as an intermediate step to obtain a parametric model or for a model selection.

It should be noted that the kernel can be interpreted in two ways: from the perspective of function estimation and of data uncertainty representation. First, when we want to approximate a function output \( y^* \) (e.g., the probability of exceeding a damage state for a fragility function or any other generic function) for the input value of \( x^* \), which we may or may not have observed, one way to estimate \( y^* \) is to compute the weighted average of the nearby observations. Here, the kernel smoothing is a systematic way of assigning the weights on the basis of how near the observations are from \( x^* \). The shape and the bandwidth, \( h \), of the kernel determine how fast the weight decays with the distance between the observation and \( x^* \). Therefore, in the case of the Gaussian kernel, we can visualize the kernel as the Gaussian PDF function centered at \( x^* \) with standard deviation \( h \) as shown in Figure 2. The weight for each observation \( x_i \) is determined by this kernel evaluated at \( x_i \), and the standard deviation will depend on the specific application of interest.

![Figure 2. Example of kernel smoothing](image)

The second approach interprets the kernel as the uncertainty of the observation, equivalent to the degree of belief for or the assumed distribution of the true value. Therefore, the kernel can be represented as a Gaussian PDF centered at each observation with a specific standard deviation \( h \) as shown in Figure 1. This standard deviation can be determined by the uncertainty of the data and it needs not be identical for each observation. These two approaches are equivalent because the kernel is a function of the absolute difference between \( x^* \) and \( x_i \).

There are three methods using the GKS approach for different types of damage measure and consideration of uncertainties: one dimensional Gaussian kernel for discrete damage states, one dimensional Gaussian kernel for continuous damage measure, and two dimensional Gaussian kernel for continuous damage and intensity measures. Since the kernel smoothing is a non-parametric method, the resulting fragility function does not have a simple parametric description. Therefore, a conventional function can be fitted for convenience using maximum likelihood method or Bayesian method. The lognormal CDF is used in conventional fragility functions, but other functions, such as the beta CDF and the truncated normal CDF, can also be used depending on the characteristics of the data. The three GKS methods are summarized in Table 1 and described in detail in the following subsections.

### Table 1. Summary of three methods for probabilistic mapping between the DSF and the SDR

<table>
<thead>
<tr>
<th>Methods</th>
<th>Outcome</th>
<th>Advantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-D GKS</td>
<td>Fragility function ((\text{Prob}(\text{DS} \geq \text{DS}</td>
<td>\text{IM}=\text{im})))</td>
</tr>
<tr>
<td>1-D GKS</td>
<td>Conditional mean and standard deviation ((\mu_{\text{DM} \mid \text{IM}}, \sigma_{\text{DM} \mid \text{IM}}))</td>
<td>Beneficial for continuous damage measure. Considers uncertainty in IM measurement.</td>
</tr>
<tr>
<td>2-D GKS</td>
<td>Conditional probability of DM given IM ((\text{Prob}(\text{DM}=\text{dm}</td>
<td>\text{IM}=\text{im})))</td>
</tr>
</tbody>
</table>

#### 2.1 One-dimensional Gaussian kernel smoothing for discrete damage states, DS

Damage states (DS) are discrete variables most often defined as ‘no damage,’ ‘slight damage,’ ‘moderate damage,’ and ‘severe damage.’ Alternatively, each damage state \( (\text{DS}_i) \) can be defined by a range of continuous DM. Thus, a set of threshold values of DM is specified for each damage state \( \text{DS}_i \) as follows:

\[
\text{DamageState} = \begin{cases} 
\text{DS}_0 & \text{if } \text{DM}_0 \leq \text{DM} < \text{DM}_1 \\
\text{DS}_1 & \text{if } \text{DM}_1 \leq \text{DM} < \text{DM}_2 \\
\vdots \\
\text{DS}_n & \text{if } \text{DM}_n \leq \text{DM} < \text{DM}_{n+1} 
\end{cases} \tag{3}
\]
where DMs are monotonically increasing threshold values for increasing i’s, and n is the number of damage states. Similarly, FEMA 356 uses four different damage states, namely, operational, immediate occupancy, life safety, and collapse prevention. Using this definition of damage states and of the conditional probability, the fragility function, \( G_i(\text{im}) = \text{Prob}(\text{DS} \geq DS_i | \text{IM} = \text{im}) \) can be rewritten as

\[
G_i(\text{im}) = \frac{\text{Prob}(\text{DS} \geq DS_i, \text{IM} = \text{im})}{\text{Prob}(\text{IM} = \text{im})}
\]

Using the kernel density estimation in Equation 1, the empirical fragility function for damage state \( i \) given the ground motion intensity \( \text{im} \) is defined as

\[
\hat{G}_i(\text{im}) = \frac{1}{n} \sum_{q=1}^{n} I(\text{ds}_q \geq DS_i) \times I(\text{im}_q = \text{im})
\]

\[
= \frac{1}{n} \sum_{q=1}^{n} I(\text{im}_q = \text{im})
\]

\[
= \frac{1}{n} \sum_{q=1}^{n} K(\frac{\text{im}_q - \text{im}}{h})
\]

\[
= \sum_{q=1}^{n} K(\frac{\text{im}_q - \text{im}}{h})
\]

where \( I(x) \) is an indicator function that is 1 if \( x \) is true and 0 otherwise. Note that this method considers the uncertainty in the IM measurements but not in the measurements of damage state.

### 2.2 One-dimensional Gaussian kernel smoothing for continuous damage measure, DM

This method estimates the conditional mean and the variance of the DM given the IM (\( \mu_{\text{DM|IM}} \), and \( \sigma^2_{\text{DM|IM}} \), respectively) and then fits a lognormal function. In other words, we can obtain the conditional probability distribution of the DM given the IM. This method is particularly useful when the damage measure is a continuous value and/or when the conditional density function of the DM needs to be convoluted with other conditional density function for further risk analysis, and thus the full conditional distribution of the DM given IM is necessary. The estimates of the conditional mean, \( \hat{\mu}_{\text{DM|IM}} \), and the conditional variance, \( \hat{\sigma}^2_{\text{DM|IM}} \), for the IM value of \( \text{im} \) can be computed using a kernel as follows:

\[
\hat{\mu}_{\text{DM|im}} = \frac{\sum_{m=1}^{M} \text{dm}_m \times K(\frac{\text{im} - \text{im}_m}{h})}{\sum_{m=1}^{M} K(\frac{\text{im} - \text{im}_m}{h})}
\]

\[
\hat{\sigma}^2_{\text{DM|im}} = \frac{\sum_{m=1}^{M} (\text{dm}_m - \hat{\mu}_{\text{DM|im}})^2 \times K(\frac{\text{im} - \text{im}_m}{h})}{\sum_{m=1}^{M} K(\frac{\text{im} - \text{im}_m}{h})}
\]

Once the mean and the variance are computed, the lognormal distribution function can be fitted to the conditional distribution of the DM given the IM by the method of moments. In other words, the obtained mean and variance are used as the mean and the variance of the lognormal distribution, and the corresponding parameters are obtained.

### 2.3 Two-dimensional Gaussian kernel smoothing for continuous damage measure, DM

Alternatively, we can estimate the conditional probability of the DM given the IM using a two-dimensional kernel as follows:

\[
\text{Prob}(\text{DM} = \text{dm} | \text{IM} = \text{im}) \equiv \frac{1}{L} \sum_{x=1}^{L} \frac{1}{h_1 h_2} K(\frac{\text{im} - \text{im}_x}{h_1}, \frac{\text{dm} - \text{dm}_x}{h_2})
\]

where \( K(x, y) \) is a two-dimensional kernel centered at \((x, y)\). This equation follows directly from the definition of the conditional probability and the kernel density estimation in Equations 1 and 4. For Gaussian kernel, \( K(x, y) \) is a bivariate standard Gaussian probability density function. In this formulation, it is assumed that the measurement errors for DM and IM are uncorrelated. If the two-dimensional kernel \( K(\text{im}, \text{dm}) \) can be factorized into \( K(\text{im}) \) and \( K(\text{dm}) \), then the previous equation can be rewritten as:

\[
\text{Prob}(\text{DM} = \text{dm} | \text{IM} = \text{im}) \equiv \sum_{x=1}^{N} \frac{1}{h_1} K(\frac{\text{im} - \text{im}_x}{h_1}, \frac{\text{dm} - \text{dm}_x}{h_2})
\]

For convenience, we can fit a conventional PDF to this empirical conditional probability distribution by minimizing the fitting error, such as a root-mean-square error (RMSE). The lognormal distribution is appropriate for this conditional probability because the IM values are bounded by zero on the lower side. One of the main advantages of this method is
that we can directly compute the conditional probability of the DM given the IM without using the second moment approximation as before. In addition, this method considers the uncertainty in both the IM and the DM measurements unlike the previous method that considers the uncertainty of only the IM by using the one-dimensional kernel.

3 APPLICATIONS

The three kernel smoothing techniques introduced in section 2 are applied to two sets of data, one from numerical simulation of a four-story steel moment resisting frame model and the other from field observations after the 2010 Haiti Earthquake.

3.1 Empirical fragility function using field observations from the 2010 Haiti Earthquake

Table 2 shows an example of the Haiti earthquake damage assessment data. These data are samples from the field-based damage assessment conducted in 2010-2011 following the earthquake in Haiti in 2010 (Ministere des Travaux Publicques, Transports et Communications 2010). Led by the Haitian Ministry of Public Works, the evaluation was conducted by civil engineers and architects having been trained in the ATC-20 damage assessment methodology (ATC-20 1989). Each building was evaluated and its GPS coordinate recorded. While a large amount of information was collected for each building, this table summarizes the most relevant fields. Damage was categorized by assigning a damage state corresponding to a central damage factor as per ATC-13 (ATC 1985). The modified Mercalli intensity (MMI) scale values obtained from the USGS (USGS shakemap 2010) were used as the ground motion intensity measure. Since no ground-motion recording equipment was present, the intensity measure was obtained from ground-motion prediction models as well as the “did you feel it” tool.

Table 2. Surveyed data after the 2010 Haiti earthquake

<table>
<thead>
<tr>
<th>ID</th>
<th>Longitude</th>
<th>Latitude</th>
<th>Central Damage Factor (%)</th>
<th>Structural Type</th>
<th># Floors</th>
<th>MMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>7754</td>
<td>-72.3464</td>
<td>18.5422</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>8.97</td>
</tr>
<tr>
<td>7755</td>
<td>-72.3463</td>
<td>18.5422</td>
<td>20</td>
<td>3</td>
<td>2</td>
<td>8.97</td>
</tr>
<tr>
<td>7758</td>
<td>-72.3461</td>
<td>18.5421</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>8.97</td>
</tr>
<tr>
<td>7759</td>
<td>-72.3460</td>
<td>18.5421</td>
<td>45</td>
<td>3</td>
<td>2</td>
<td>8.07</td>
</tr>
<tr>
<td>7760</td>
<td>-72.3459</td>
<td>18.5421</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>8.07</td>
</tr>
</tbody>
</table>

The scatter plot of the data is shown in Figure 3(a). This figure shows the distribution of the data collected from Haiti, as described in Table 2. The samples at each MMI level and the corresponding mean damage factor are shown as a circle whose size is proportional to the number of buildings at that MMI level. Over 250,000 buildings are included in this figure.

3.1.1 Data binning method

Data binning is often used to identify trends or even to develop non-parametric regression curves. The data are divided into bins of ground motion intensity and the damage averaged over that bin. Binning however introduces additional error to the data. One of the main issues with data binning for fitting a fragility curve is that the resulting curve is very sensitive to the subjective choice of binning. The curves in Figure 3(b) and (c) have the same bin size, but they are shifted, resulting in very different curves. Similarly, reducing the bin size also results in very different results, as shown in Figure 3(d). The figure also demonstrates two other common disadvantages of using binned average to fit fragility curves. When binning, all IM values within a bin are considered the same. Hence, in Figure 3(a) the data at MMI levels 6.6 and 7.4 are lumped together to create Figure 3(b), assigning them an MMI level of 7.0. Furthermore, damage measures are computed only at each bin, resulting in sparse representation of fragility curves. For instance, Figure 3(d) shows a gap at MMI levels 8.25-8.75, for which no data exists. For these reasons, Gaussian Kernel Smoothing is recommended as a means to develop fragility curves which are smooth and maximize the use of all data points, as explained in section 2.

3.1.2 Kernel smoothing method

Figure 4(a) shows the mean damage factors introduced in section 3.1.1. In this figure, the mean damage factor at every IM level is obtained by weighting every data point by the Gaussian kernel and represented as a dash line. As can be seen from Figure 4(a) and (b), the choice of kernel range, or the vari-
ance of the Gaussian function, influences the resulting curve, yet the influence is primarily on the smoothness of the curve rather than its shape. The kernel method can also be used to obtain the standard deviation of the fragility function, as seen in Figure 4(c). Figure 4(d) demonstrates that a continuous curve can still be created using the kernel smoothing method even when there is a large data gap, which was not possible using binning as shown in Figure 3(d). Finally, we note that kernel smoothing results in significantly smoother curves than those obtained from binning. Table 3 summarizes the advantages of using the kernel method.

Table 3. Advantages of the Gaussian kernel density estimation in comparison to the data binning method

<table>
<thead>
<tr>
<th>Kernel Density Estimation</th>
<th>Data Binning Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>All the available {im, dm} pairs are used with different weights to estimate the fragility function for damage state j, (\hat{G}(im)). Thus, this method reduces problems caused by lack of {im, dm} data or biased sampling.</td>
<td>Lack of data or biased data can result in some bins with no data - (\hat{G}(im)) values cannot be defined for these bins (see Figure 3(d)).</td>
</tr>
<tr>
<td>(\hat{G}(im)) values can be computed at all the values of IM, thus resulting in a dense representation of (\hat{G}(im)). Also, all IM values are considered as they are measured.</td>
<td>(\hat{G}(im)) values can be computed only at each bin, resulting in a sparse representation of (\hat{G}(im)) (see Figure 3(b) and (c)). All the IM values within a bin are considered the same (i.e., they are treated as the center value of the bin). This can introduce a bias in the fit (see Figure 3(a)-(c)). A small measurement error may assign a sample to a different bin, which may affect the regression results significantly.</td>
</tr>
<tr>
<td>Discretization error for IM is avoided.</td>
<td></td>
</tr>
</tbody>
</table>

The resulting fragility function is a smooth curve and the degree of smoothness can be varied to reflect the uncertainty in the data. Uncertainty in the IM and DM values in the data can be accounted for by the Kernel’s bandwidth. It is difficult to include the uncertainty in IM and DM measurements.

3.1.3 One-dimensional Gaussian kernel smoothing for discrete damage state

Equation 5 is used to obtain a fragility curves from one-dimensional Gaussian kernel smoothing of the 2010 Haiti earthquake data. Individual fragility curves can be obtained for each damage state, such as shown in Figure 6. The fragility functions correspond to the probability of experiencing or exceeding “slight” and “extreme” damage respectively.

![Figure 5. Fragility functions using the Gaussian kernel smoothing for the Haiti earthquake data](image)

3.1.4 One-dimensional Gaussian kernel smoothing with continuous damage measure

Equation 6 can be used to develop a mean damage curve based on the damage state data for each building. Equation 7 can then be used to obtain the complete damage distribution conditioned on IM. Assuming a lognormal conditional distribution ensures that damage is bounded at zero damage. Other distributions could also be used, such as the beta distributions, the moments of which can be obtained similarly by method of moments. Figure 7 below shows the mean damage curve, the mean plus and minus one standard deviation, as well as three full distributions of damages conditioned on IM={7, 8.5 and 10}.

![Figure 7. Mean damage curve and standard deviations](image)

3.1.5 Two-dimensional Gaussian kernel smoothing with continuous damage measure

Two-dimensional Gaussian kernel smoothing can be used to develop a complete joint-probability plane, giving the probability of every DM-IM pair. Assuming that the measurement error in DM and IM are independent, Equation 8 can be used to obtain the conditional probability of damage given IM. The two-dimensional GKS provides further insight into the behavior and response of structures to earthquake ground-motion. Figure 8 for instance shows that for higher IM levels, buildings do not simply shift to higher damage states, but are marked by higher rates of extreme damage. This would be con-
sistent with the expected behavior of very brittle buildings, such as those commonly found in Haiti.

Figure 6. One-dimensional Gaussian kernel smoothing example of the mean DM and conditional distribution of given IM, based on Haiti earthquake data.

Figure 7. Two-dimensional Gaussian kernel smoothing example of the mean DM and conditional distribution of damage given IM, based on Haiti earthquake data.

3.2 Analytical fragility function using a four-story steel moment resisting frame model

3.2.1 One-dimensional Gaussian kernel smoothing for discrete damage state

A set of numerically simulated data is collected from a four-story two-bay steel special moment-resisting frame (SMRF) model. This frame is a perimeter lateral resisting system of an office building designed in Los Angeles based on current seismic provisions such as the IBC and the AISC. This model was subjected to 40 ground motions, each of which is scaled to various intensities. The spectral acceleration at the first mode period ($Sa(T_1, 2\%)$), measured in g, is used as an IM of the ground motion, and the story drift ratio (SDR) is used to quantify a DM. More details about the model, data, and the analysis procedure can be found in Lignos & Krawinkler (2009) and Noh et al. (2011). Figure 8(a) and (b) shows the scatter plot of IM and DM and the fragility functions obtained using the data binning methods, and Figure 8(c) shows the fragility functions from the one-dimensional GKS method. The five discrete damage states, DS0, DS1, ..., DS4, are defined as no damage (i.e., within the elastic limit) ($0\% \leq SDR < 1\%$), slight damage ($1\% \leq SDR < 2\%$), moderate damage ($2\% \leq SDR < 3\%$), severe damage ($3\% \leq SDR < 6\%$), and collapse ($6\% \leq SDR$), respectively. The threshold values of SDR are selected as representative values to describe different damage states based on current practice (FEMA 356 and FEMA 440). We can observe that the GKS method provides a smooth and continuous fragility functions unlike the data binning method as explained in Table 3.

Figure 8. Relationship between IM and DM for the numerical simulation

3.2.2 One-dimensional Gaussian kernel smoothing with continuous damage measure

Using the continuous DM, we can obtain a more detailed relationship between IM and DM. Figure 8(d) shows the conditional mean and standard deviation of DM given IM for various IM values. Figure 9 shows the scatter plot of data and the conditional mean with one standard deviation above and below it. The right hand side plot in Figure 9 shows the conditional distribution of DM given the IM values of 0.7, 1.3, and 1.9 g. Lognormal distribution is fitted using the method of moments based on the conditional mean and standard deviation for this figure. We can observe that as the IM values increases, both the conditional mean and the variance of the DM increases.

Figure 9. Conditional probability of DM given IM using one-dimensional kernel for the numerical simulation
3.2.3 Two-dimensional Gaussian kernel smoothing with continuous damage measure

Figure 10 shows the conditional distribution of DM given IM={0.7, 1.3, and 1.9} using two-dimensional GKS method. Two-dimensional GKS method provides very detailed information about the conditional probability distribution of DM given IM as shown in Figure 11.

![Figure 10](image1)

Figure 10. Conditional probability of DM given IM using two-dimensional kernel for the numerical simulation

![Figure 11](image2)

Figure 11. Conditional probability of DM given IM using two-dimensional kernel for the numerical simulation

4 CONCLUSIONS

This paper introduces a new framework to compute empirical and analytical fragility functions using Gaussian kernel smoothing methods. The kernel smoothing methods estimate a functional relationship between two variables by taking weighed average of nearby data, and this weighting function is referred to as a kernel. Three different methods are presented in this paper to provide different levels of information for discrete and continuous damage measures. The first method uses one-dimensional kernel to compute fragility functions for discrete damage states, while the second and the third methods use one- and two-dimensional kernels, respectively, for continuous damage measures. The latter two provides more detailed information about the relationship between IM and DM because they allow us to take advantage of the continuous characteristics of DM instead of discretizing it as the conventional methods do. These methods are applied to two sets of data collected from field observations after the 2010 Haiti Earthquake and numerical simulations of a four-story steel special moment-resisting frame. The results show that the kernel-based methods can reduce the bias due to discretization and provide more detailed information about the probabilistic relationship between DM and IM.

REFERENCES


