On the use of wavelet coefficient energy for structural damage diagnosis

H. Noh & A. S. Kiremidjian

Department of Civil and Environmental Engineering, Stanford University, CA, 94305

ABSTRACT: The response motions of a reinforced concrete column subjected to a series of scaled strong motion of 1994 Northridge earthquake on University of Nevada, Reno shake table are analyzed in order to determine the damage status of the column. Since earthquake motion is non-stationary, wavelet analysis using Morlet wavelet is applied to model the response signal. Wavelet coefficient energy at a scale defined by Nair & Kiremidjian (2007) as well as wavelet coefficient energy at a time are extracted from output acceleration signals and compared to one another to detect damage on the column. Also, the effective time of vibration is defined to analyze the result. The results show that as the damage extent increases the wavelet coefficient energies at low scales (high frequency) decrease while those at high scales (low frequency) increase. Also, the time history of the wavelet coefficient energies spread out wider for more severely damaged cases.

1 INTRODUCTION

Efficient and reliable estimation of damage immediately after a large earthquake is of great importance for reducing potential injuries and business interruption. Previous work on damage detection has focused on the use of ambient vibrations obtained before the occurrence of an earthquake and immediately after such an event. In this paper, the response motion obtained during an earthquake is used to determine if a change has occurred due to the strong shaking. Since earthquake motion is non-stationary, previously used autoregressive models for damage detection do not apply. However, use of wavelet coefficient energies of the input and the output vibration signals is a particularly suitable approach for detecting damage in the structure from earthquake strong motion.

Early work on using wavelet analysis for structural health monitoring has been carried out in several different perspectives. Ghanem & Romeo (2000) represented the equation of motion in terms of wavelet basis and solved the inverse problem of time-varying system. Kijewski & Kareem (2003) used wavelet analysis for system identification, and Hou et al. (2000) detected sudden changes in a signal using discrete wavelet transform. The model that will be described in the paper uses the Morlet wavelet to characterize the earthquake motion. The energy of the wavelet coefficients at a particular scale is defined as the square of the Euclidean norm of the wavelet coefficients vector as defined by Nair & Kiremidjian (2007). Similarly the energy of the wavelet coefficients at a time is defined. The model is tested with acceleration data for a series of tests performed on a bridge column subjected to successively increasing earthquake ground motion applied by the University of Nevada, Reno (UNR) shake table. In the following sections the description of the experiment is presented followed by the description of algorithm.

2 DESCRIPTION OF EXPERIMENT

Four large-scale circular reinforced concrete bridge columns are designed, constructed, and tested at University of Nevada, Reno (UNR) Large Scale Structures Laboratory (LSSL) in order to investigate the effects of near-fault ground motion and fault-rupture on the seismic response of typical reinforced concrete bridge columns (Choi et al. 2007). The columns have different initial periods and are tested under simulated horizontal earthquake motions in the fault normal direction. First two columns are designed based on the 2004 Caltrans Seismic Design Criteria (SDC) version 1.3, to satisfy the current Caltrans near-fault provisions with a target displacement ductility capacity or 5. A new design spectrum was developed from the first two column test, and other two columns are designed based on the new design spectrum.

The seismic responses of ETN column which belongs to the first group of columns are used for analysis in this paper. ETN stands for extra tall near-
fault column. The height of ETN is 108.5 in. and the clear height is 98.5 in. The diameter of the specimen is 14 in. and the aspect ratio is 7.75. The initial period calculated based on cracked stiffness for ETN is 1.5 second. 22 #4 grade 60 rebars are used for longitudinal reinforcement, and galvanized steel wire with the diameter of 0.25 in. is used for transverse reinforcement with 1.0 in. of pitch. Figure 1 shows the drawing of ETN column. Various sensors are deployed on the column and the reinforcement to measure strain, curvature, displacement, acceleration, and force during the test. For the analysis, acceleration data are used. A crossbow CXLOZLF1 accelerometer is used to measure the acceleration at the top of the column as the output vibration signal. The acceleration of the shake table is also used as the input vibration signal for the analysis.

For the test, the specimen is centered on the shake table, and the footing is securely attached to the shake table. The mass rig system is connected to the top of the specimen to create a total inertial mass of the column of 62 kips. Two 4 x 4 x 8 ft. concrete blocks where each block weighs about 20 kips are used as the mass rig system. A steel spreader beam is bolted to the top of the column head to provide an axial load of 62 kips to the column. Figure 2 shows the shake table setup. A series of the fault normal component of the acceleration record at the Rinaldi Receiving Station recorded during the 1994 Northridge earthquake is applied to the column through the shake table. The amplitude of the input acceleration is scaled by an increasing factor of 0.05, 0.10, 0.20, 0.30, 0.45, 0.60, 0.75, 0.90, 1.05, 1.20, 1.35, 1.50, and 1.65 in the series of the tests. These tests will be referred as damage pattern (DP) 1, 2 … 13 hereafter. The column is loaded in the north-south direction. As the test progresses visible damage on the column is recorded as well as the measurements from the sensors.

3 ALGORITHM

In this algorithm, wavelet analysis is used to detect damage occurred during the earthquake. Damage detection is based on the premise that damage in the structure will cause changes in dynamics of the structure which is reflected on the vibration measurements of the structure. While Fourier analysis and autoregressive analysis are good for analyzing stationary signals, wavelet analysis allows us to look at the signal in time-scale domain which can represent the time-varying characteristics of the frequency components of the signal. Thus it is adequate to use wavelet analysis to study earthquake strong motion responses which are highly non-stationary. For that purpose wavelet transform of response signals are computed, and the wavelet coefficient energy at a scale ($E_{scale(a)}$) defined by Nair & Kiremidjian (2007) as well as the wavelet coefficient energy at a time ($E_{shift(b)}$) is used as a damage sensitive feature (DSF) of each signal. It is proven by Nair & Kiremidjian (2007) that $E_{scale(a)}$ of the acceleration signal at high scale is well correlated with the damage extent of the structure assuming that the damaged structure is an equivalent linear system with reduced stiffness. The wavelet coefficient energy at various scales are compared between different response signals in order to determine the damage status. In the model, it is hypothesized that an initial recording from a strong but a non-damaging earthquake has occurred at some point prior to a damaging event providing baseline energies for that structure. DP 1 did not cause any damage to the column and thus is used as the baseline. Also, both the input and the output vibration signals are available. With larger events that cause damage to the structure a migration of these energies can be observed, and changes in

![Figure 1. ETN column with dimensions (modified from Choi, 2007)](image-url)

![Figure 2. Shake table setup (modified from Choi, 2007)](image-url)
them are found to be indicative of the onset of damage. Detailed description of the algorithm is presented in the following sub-sections.

### 3.1 Data Correction

Before taking the wavelet transform of the signals, the signals are corrected by subtracting the mean of the beginning part of the signal where the strong motion has not started yet and dividing by the root-mean-square (RMS) value of the mean corrected input signals. Theoretically, the RMS values of the input signals should be proportional to the scale factor of the input acceleration, but due to the noise they are not exactly the same. Table 1 shows the RMS value of each input accelerations as well as the description of damage for each DP. The purpose of subtracting the mean is to offset the different initial conditions of the sensors, and that of dividing by RMS value of the input acceleration is to reduce the effect of different loading conditions. After the correction is performed the wavelet transform of the whole signals are computed.

<table>
<thead>
<tr>
<th>DP</th>
<th>Scaling factor</th>
<th>Input RMS value</th>
<th>Description of Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.0053</td>
<td>no damage</td>
</tr>
<tr>
<td>2</td>
<td>0.10</td>
<td>0.0105</td>
<td>small cracks on south side</td>
</tr>
<tr>
<td>3</td>
<td>0.20</td>
<td>0.0210</td>
<td>more cracks on both sides</td>
</tr>
<tr>
<td>4</td>
<td>0.30</td>
<td>0.0313</td>
<td>more cracks</td>
</tr>
<tr>
<td>5</td>
<td>0.45</td>
<td>0.0448</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0.60</td>
<td>0.0576</td>
<td>spalling on south side</td>
</tr>
<tr>
<td>7</td>
<td>0.75</td>
<td>0.0696</td>
<td>more spalling, cracks open wider</td>
</tr>
<tr>
<td>8</td>
<td>0.90</td>
<td>0.0827</td>
<td>more spalling, cracks open wider</td>
</tr>
<tr>
<td>9</td>
<td>1.05</td>
<td>0.0963</td>
<td>spalling on north side</td>
</tr>
<tr>
<td>10</td>
<td>1.20</td>
<td>0.1094</td>
<td>5 spirals exposure</td>
</tr>
<tr>
<td>11</td>
<td>1.35</td>
<td>0.1224</td>
<td>more spirals exposure</td>
</tr>
<tr>
<td>12</td>
<td>1.50</td>
<td>0.1352</td>
<td>longitudinal bar exposure</td>
</tr>
<tr>
<td>13</td>
<td>1.65</td>
<td>0.1478</td>
<td>bar exposure on both sides</td>
</tr>
</tbody>
</table>

### 3.2 Wavelet Transform

The continuous wavelet transform of a function \( f(t) \in L^2(\mathbb{R}) \), where \( L^2(\mathbb{R}) \) is the set of square integrable functions is defined by:

\[
Wf(a,b) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{a}} \psi^*(\frac{t-b}{a}) dt
\]

where, \( \psi(t) \in L^2(\mathbb{R}) \) is called the mother wavelet and \( * \) represents complex conjugate. Wavelets are localized waves that span a finite time duration. The mother wavelet \( \psi(t) \) is dilated by scaling parameter \( a \) and translated by shift parameter \( b \) to create basis functions called daughter wavelets. The basis functions are convoluted with a function to compute the wavelet coefficients \( Wf(a,b) \). For this analysis, Morlet wavelet is used as a mother wavelet since it has a closed form expression and its shape resembles earthquake pulses. The expression for the Morlet wavelet used in this paper is given as:

\[
\psi(x) = e^{-\frac{x^2}{2}} \cos(5x)
\]

Figure 3 shows the Morlet wavelet.

Wavelet coefficients of response signals from different inputs clearly show different patterns as represented in Figure 4. As the intensity of the input motion increases, the peaks of wavelet coefficients shift both in time and in scale. High intensity of the input signal is strongly correlated with the intensity of damage on the structure, thus the changes in the pattern of wavelet coefficients is well correlated with the damage status of the structure. This pattern...
is extracted out as DSF using the wavelet coefficient energy.

3.3 Feature Extraction

3.3.1 Wavelet Coefficient Energy at scale \( a \)
The shift in scale of the wavelet coefficients is represented through the wavelet coefficient energy at a scale, \( E_{\text{scale}(a)} \), defined by Nair & Kiremidjian. Once the wavelet coefficients are ready, wavelet coefficient energy at scale \( a \) is computed for each output signal as DSF. It is given as:

\[
E_{\text{scale}(a)} = \sum_{b=1}^{K} |Wf(a,b)|^2
\]

where, \( K \) is the number of data points in the signal and \( |\cdot| \) is the absolute value of the quantity. The square of the wavelet coefficient is also called the scalogram. It is found that the energies of the fifth, sixth and seventh dyadic scales of the Morlet wavelet are the largest among all scales, and changes in the structure appear to be manifested as changes in these energies. In order to compare \( E_{\text{scale}(a)} \)’s for the acceleration outputs from different input motions \( E_{\text{scale}(a)} \)’s are normalized by the sum of \( E_{\text{scale}(a)} \)’s at fifth, sixth, and seventh dyadic scales computed for each input motion. The results are presented in the following section.

3.3.2 Wavelet Coefficient Energy outside Effective Time of Vibration

For capturing the shift in time of the peaks of wavelet coefficients, the summation of the scalogram over the scaling parameter \( a \) is defined as the wavelet coefficient energy at time \( b \). This is given by:

\[
E_{\text{shift}(b)} = \sum_{a=2}^{7} |Wf(a,b)|^2
\]

Only the fifth, sixth, and seventh dyadic scales are considered in the equation above for the same reason as in \( E_{\text{scale}(a)} \). It is shown in Figure 5 that time history of \( E_{\text{shift}(b)} \) flattens and decays slower as the intensity of the input motion increases. This observation is consistent with the time shift of peaks of wavelet coefficients in Figure 4. Figure 6 shows the cumulative sum of \( E_{\text{shift}(b)} \) of input and output response for different DP’s. Since the input signals are scaled versions of the same signal and normalized by their RMS values, the cumulative sums of \( E_{\text{shift}(b)} \)’s of input signals are almost identical to each other. Those of output signals, however, decrease in the magnitudes and in the slope as the damage progresses. It also shows the flattening of \( E_{\text{shift}(b)} \) as the damage extent increases.

Figure 5. \( E_{\text{shift}(b)} \) for different DP’s: (a) DP 1; (b) DP 3; (c) DP 5; (d) DP 7; (e) DP 9; (f) DP 11; (g) DP 13

Figure 6. Cumulative sum of \( E_{\text{shift}(b)} \) for different DP’s: (a) DP 1; (b) DP 3; (c) DP 5; (d) DP 7; (e) DP 9; (f) DP 11; (g) DP 13

In order to estimate how fast \( E_{\text{shift}(b)} \) is decaying, effective time of vibration for the baseline is defined and the percentage of \( E_{\text{shift}(b)} \) of other output signals outside the effective time is computed. Effective time of vibration is defined as the time between \( t_{05} \) and \( t_{95} \) where \( t_{05} \) and \( t_{95} \) are the times when the cu-
mulative sum of $E_{\text{shift}(b)}$ are 5% and 95% of the total sum of $E_{\text{shift}(b)}$, respectively. If the time history of $E_{\text{shift}(b)}$ for a response signal decays slower than that of baseline, the percentage of $E_{\text{shift}(b)}$ for the response signal outside the effective time of vibration for the baseline would be higher than 0.1. The percentage will increase as the time history of $E_{\text{shift}(b)}$ decays slower. Thus, the percentage of $E_{\text{shift}(b)}$ outside the effective time of baseline vibration is defined as the second DSF. The results are shown in the following section.

4 RESULTS

Figure 7 shows the normalized $E_{\text{scale(a)}}$’s at fifth, sixth, and seventh dyadic scales. When the column is not damaged $E_{\text{scale(a)}}$ at sixth dyadic scale is dominant. As the intensity of input acceleration grows and the damage progresses, the fraction of $E_{\text{scale(a)}}$ at sixth dyadic scale decreases while that at seventh dyadic scale increases. $E_{\text{scale(a)}}$ at fifth dyadic scale monotonically decreases. The fifth, sixth and seventh dyadic scale of Morlet wavelet with 100 Hz sampling frequency correspond to the pseudo-frequency of 2.5391 Hz, 1.2695 Hz, and 0.6348 Hz, respectively, assuming the center frequency of 0.8125 Hz for the Morlet wavelet. Scales are inversely related to pseudo-frequencies. Thus, the result indicates that as the damage progresses high frequency component of the response decreases while low frequency component of the response increases. According to the experiment report (Choi et al., 2007), spalling of concrete starts at DP 5 and the strain exceeds its yield strain at DP 4. This is represented as the consistent values of $E_{\text{scale(a)}}$ at sixth dyadic scale up to DP 4 and its decrease afterwards. Similar pattern is observed for the instantaneous frequency of each signal. Instantaneous frequency at time $t$ is defined as the pseudo frequency of the scale which has the maximum wavelet coefficients among all the scales at time $t$. Thus, the instantaneous frequency is the dominant frequency of the signal at time $t$. Figure 8 shows how the instantaneous frequency decreases as the damage extent increases. The instantaneous frequencies between $t_{05}$ and $t_{05}$ are fairly constant with several outliers, thus they are averaged out for Figure 8. Further experiments and research are necessary to determine the threshold values of $E_{\text{scale(a)}}$’s at sixth and seventh dyadic scale for damage detection.

The result of $E_{\text{shift}(b)}$ outside the effective time of vibration is presented in Figure 9. As expected the percentage of $E_{\text{shift}(b)}$ outside the effective time of vibration is close to 10% for lower DP’s and increases as the damage increases. The result is consistent with the previous result that up to DP 4 the percentage is low and it increases afterwards and after DP 9 the percentage stabilizes. The result implies that as the damage progresses the response motion decays slower which can be correlated with lighter damping. Further experiments and research are necessary to determine the threshold value of $E_{\text{shift}(b)}$ outside the effective time of vibration for damage detection.
CONCLUSION

In this paper, wavelet transform of strong motion response is studied and the energies of wavelet coefficient are defined for damage detection. The algorithm is applied to the acceleration data from reinforced concrete bridge column test subjected to a series of scaled 1994 Northridge earthquake motion performed at University of Nevada, Reno. The response from the smallest excitation that caused no damage to the column is assumed as the baseline case. The peaks of wavelet coefficients for each response shift to higher scale and later time as the damage progresses. These characteristics are reflected in the DSF’s defined as the sum of scalogram for each scale ($E_{scale(a)}$) and each time($E_{shift(b)}$).

The result shows that $E_{scale(a)}$'s for lower scales (i.e. higher frequency components in signal) decrease as the damage extent increases while that for higher scale (i.e. lower frequency components in signal) increases. Also, $E_{shift(b)}$ outside the effective time of baseline vibration increases as the damage extent increases which indicates that the response of the column damps out slower with more damage. $DP$ where these changes in DSF’s happen is around $DP$ 4 where the concrete starts spalling according to the damage description by Choi et al. (2007). Further testing and research are necessary to validate the algorithm and set the thresholds of DSF’s to the extent of damage reflected by the various damage states. Significantly more research is needed to be able to localize the damage based on the obtained measurements.

REFERENCES


AKNOWLEDGEMENTS

The research presented in this paper is partially supported by the National Science Foundation Research Grant and the Samsung Graduate Fellowship. We are most appreciative of their support.