Sequential structural damage diagnosis algorithm using a change point detection method

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A B S T R A C T

This paper introduces a damage diagnosis algorithm for civil structures that uses a sequential change point detection method. The general change point detection method uses the known pre- and post-damage feature distributions to perform a sequential hypothesis test. In practice, however, the post-damage distribution is unlikely to be known a priori, unless we are looking for a known specific type of damage. Therefore, we introduce an additional algorithm that estimates and updates this distribution as data are collected using the maximum likelihood and the Bayesian methods. We also applied an approximate method to reduce the computation load and memory requirement associated with the estimation.

The algorithm is validated using a set of experimental data collected from a four-story steel special moment-resisting frame and multiple sets of simulated data. Various features of different dimensions have been explored, and the algorithm was able to identify damage, particularly when it uses multidimensional damage sensitive features and lower false alarm rates, with a known post-damage feature distribution. For unknown feature distribution cases, the post-damage distribution was consistently estimated and the detection delays were only a few time steps longer than the delays from the general method that assumes we know the post-damage feature distribution. We confirmed that the Bayesian method is particularly efficient in declaring damage with minimal memory requirement, but the maximum likelihood method provides an insightful heuristic approach.

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1. Introduction

Structural health monitoring (SHM) provides a great potential for civil structures in providing safe and functional built environment by diagnosing their damage efficiently and reliably. In the past few decades, there has been a great effort to develop the hardware aspects of SHM [1–3]; however, we still need to develop robust and reliable algorithms that can take a full advantage of the collected data. The challenges for properly analyzing civil structures come from the fact that civil structures are complex systems with various materials and complicated geometry. In addition, they are subjected to significant uncertainties in loading and environmental conditions. Thus, identifying the features that are sensitive to damage but insensitive to environmental and loading conditions is difficult, and the features inherently contain uncertainties because
of the aforementioned reasons and measurement errors. Therefore, we need to develop algorithms that appropriately consider the uncertainties of the features.

In this paper, we introduce a vibration-based damage diagnosis algorithm that sequentially processes data while considering uncertainties of individual features and time of damage in an efficient way. This algorithm uses time-series based damage sensitive features (DSFs) that are computationally inexpensive to extract from structural responses. As a result, it is particularly suitable to be used in wireless sensing units. In the past decade, various time-series based features have been developed for damage diagnosis algorithms [4–11]. Among these numerous options, we use the coefficients of both the singlevariate and multivariate autoregressive (AR) and autoregressive with the exogenous input (ARX) models as DSFs because of their relationship to structural parameters [9] and their computational efficiency. Moreover, the use of the multivariate ARX model enables us to combine the information collected from multiple sensors.

The other important element of the algorithm is the change point detection method [12], which sequentially receives features extracted from structural vibration data and conducts a hypothesis test. By combining a sequence of weak or noisy evidences, the resulting decision can become more reliable. The test involves declaring damage on the basis of the allowable false alarm rate and the posterior distribution of the time of damage. This distribution is computed using the pre- and post-damage feature distributions and the prior distribution of the time of damage. Consequently, the algorithm considers uncertainties of both the feature and the time of damage and systematically combines the information from the previously collected DSFs, instead of relying on decisions based on evaluating a single sample. This is important because individual DSF value is a weak or noisy damage indicator but the combination of them over time may become a strong evidence of damage. In addition, unlike the previous damage diagnosis algorithm that uses the change point detection method [13], which assumes that the pre- and post-damage feature distributions are known a priori, the proposed algorithm assumes that the post-damage feature distribution is unknown and estimates it using the collected data, which is more applicable in practice.

Change point detection method has been applied to a variety of fields for rapid detection of anomaly, including computer network [14,15], manufacturing quality control [16], and environment monitoring [17,18], but it has not been applied for SHM purposes. The most relevant method is a sequential probability ratio test (SPRT), where data is sequentially subjected to a statistical test about their distribution as they are collected. We can set constraints on the upper bounds for false positive and false negative error rates. Among many valid sequential tests, the SPRT is analytically proven to minimize the average sample number required to make a decision about the state of the data [19]. Sohn et al. [6] used the SPRT for the standard deviations of the residual of the AR model to classify structural damage, and Herzog et al. [20] used it for the residuals of the similarity-based model to monitor aircraft engines. It was also applied to monitoring nuclear power plants [21]. Despite its optimality to make a decision, the SPRT has a drawback to be applied to continuous monitoring of structures, compared to change point detection method. The SPRT assumes that the data are independent samples drawn from the same distribution (e.g., either undamaged or damaged); thus, if the sequence of data include samples from more than one distribution, (e.g., mixture of data from undamaged and damaged cases), the optimality condition is not valid and the decision making process becomes much slower. Sohn et al. [6] indicated this problem by “oscillatory behavior” where the test statistic initially moves away from the threshold when the structure is intact, and then when the damage happens it moves toward the threshold and eventually crosses the threshold but at a slower rate. Introducing some “weighting factor”, as they suggested, may mitigate the problem, but it is not clear what an optimal weighting scheme is because a proper scheme will be based on the information or likelihood of which distribution each sample follows, which is the purpose of conducting the hypothesis test in the first place. On the other hand, the change point detection method presented in this paper addresses a slightly different problem and conducts hypothesis tests about the time of change assuming that the data before the change time follows one distribution while the subsequent data are distributed differently. This approach prevents the test statistics from oscillating. The comparison of these behaviors is presented in the result section below. In addition, this method incorporates the Bayesian framework considering the uncertainty about the time of change. When prior information about the change point is not available, a diffused prior distribution can be assigned; however, if we know the prior distribution, for example from risk analysis, the Bayesian framework can enhance the decision process by properly incorporating this information. Finally, the most important and innovative aspect of our algorithm is that it can detect damage without knowing the post-damage distribution, which is not feasible in other sequential damage detection methods.

In summary, our contributions include the followings: First, we investigated various types of DSFs and explored their sensitivity to damage. Secondly, we represented the damage classification problem as the trade-off between the false alarm rate, defined as the rate of declaring damage before it actually happens, and the detection delay, that is the time incurred to assert damage after it happens [22,23]. The third contribution is the development of various methods to estimate and update the post-damage feature distribution, if it is unknown, as we collect data. We also developed an approximate method to reduce computation and memory load and evaluated their performance. With the estimated distribution, we can use the change point detection method to detect damage identification. Under appropriate assumptions, the change point detection method minimizes the damage detection delay for an allowed probability of falsely declaring damage using a Bayesian framework. Therefore, once we choose a false alarm rate, which sets the reliability of the algorithm, the algorithm automatically optimizes the detection delay to locate the damage both in space and time. For unknown post-damage feature distribution cases, our algorithm approximates this optimal detection delay closely by updating the estimation of the feature distribution every time a new DSF is collected. Barnard [24] proposed the idea to use the maximum likelihood method to detect mean shifts of data with known variance when the amount of shift is unknown, and Siegmund and Venkatraman [25] solved the statistical properties of this procedure. Our algorithm incorporates the Bayesian framework to the problem by assigning a prior distribution to the time of damage and reformulate the procedure using the two approaches to estimate the unknown distribution: the maximum likelihood method and the Bayesian method.
We verified this algorithm using a set of experimental data obtained from a series of shake table tests and multiple sets of simulated data. The experiment involved the collection of ambient-vibration response data from a four-story steel moment-resisting frame for multiple damage scenarios. The simulation involved the generation of a set of 10,000 time-series that consist of random numbers whose distribution changes during the simulation in order to further test the consistency and the robustness of the algorithm after experimental validation. The DSF is extracted from each data, which is sequentially processed using the proposed algorithm to estimate the post-damage feature distribution, if unknown, and then the change point detection method to identify damage with minimal delay within the given false alarm rate. The performance of this algorithm is compared with that of the SPRT using the experimental data.

This paper is organized as follows. The algorithm section defines various DSFs using the single-variate and multivariate AR and ARX models and describes the conventional hypothesis test, sequential change point detection method, and the new method to estimate the post-damage feature distribution when it is unknown. The validation section describes the simulation and the shake table experiment and the application results. Finally, the conclusion section summarizes the paper and discusses limitations of the algorithm and future directions.

2. Algorithm

The algorithm consists of three steps: (i) collection of structural responses, (ii) extraction of a damage sensitive feature (DSF), and (iii) identification and classification of damage. Fig. 1 provides the summary of the algorithm. The first step involves sequentially obtaining structural responses from multiple sensors and normalizing them. The data normalization includes subtracting the mean from each data set and then dividing by its standard deviation. In the second step, we apply the single-variate AR and multivariate ARX models to extract the DSF. In this study, various combinations of the AR and ARX coefficients are defined as DSFs because they are closely related to structural parameters as derived by Nair et al. [9] and sensitive to damage as shown in [26,13]. Nair et al. [27] also showed that the distribution of the first AR coefficients follows a Gaussian mixture model. The algorithm then defines a score that involves the distributions of the DSF before and after the damage. When the post-damage distribution of the DSF is unknown, two methods that use the maximum likelihood estimation and the Bayesian updating scheme are applied to estimate the score, and an approximate method is developed to compute the score efficiently. The change point detection method essentially performs a sequential hypothesis test to identify damage, which is equivalent to a threshold test of the score based on the desired false alarm rate. This third step allows the algorithm to benefit from the relationship of the observed features in time when the current damage state is unknown. In this algorithm, we assume that the damage is irreversible; thus, the algorithm can eventually identify damage if it keeps updating the score long enough. The change point detection method, however, minimizes the detection time delay for a desired false alarm rate. In other words, the method makes a trade-off between the detection time delay and the false alarm rate.

Section 2.1 describes the DSF extracted from the collected acceleration responses of the structure in various damage states. Section 2.2 then introduces the sequential change point detection method that identify damage for two cases when the distribution of the DSF is known or unknown a priori for damaged cases.

2.1. Feature extraction

We focus on different combinations of the single-variate AR and multivariate ARX coefficients to generate DSFs:

2.1.1. Single-variate AR model

The AR model is a linear predictor, which predicts the system output as a linear function of previous outputs and captures the linear dynamics of ideal structures. To extract the feature, we first subtract the mean of each acceleration recording from the original recording to zero-mean, and then fit the AR model. Let $x_i(t) (i = 1, 2, ..., N)$ denote the zero-meaned acceleration

![Fig. 1. Summary of the algorithm.](image)
recordings collected from sensor $i$, where $N$ is the number of sensors. The single-variate AR model of order $p$ is given as
\begin{equation}
X_i(t) = \sum_{k=1}^{p} a_k X_i(t-k) + \epsilon_i(t)
\end{equation}
where $a_k$ is the $k$th AR coefficient and $\epsilon_i(t)$ is the residual. We investigated the performance of various combinations of these AR coefficients as DSFs, such as $\{a_1, a_2, \ldots, a_\alpha\}$. The AR modeling of the data includes the removal of trends, obtaining the optimal model order, and checking the assumptions about the residuals of the AR model. More details are given by Noh et al. [26].

2.1.2. Multivariate ARX model
The multivariate ARX model predicts a multivariate system output as a linear combination of previous outputs and exogenous inputs. We fit this model to the vector of acceleration data collected from multiple sensors at time $t$, denoted by $X(t)$. The multivariate ARX model for the input $U(t)$, output $X(t)$, and model orders of the AR and the exogenous input $\{p, q\}$ is given as
\begin{equation}
X(t) = \sum_{k=1}^{p} A^k X(t-k) + \sum_{k=1}^{q} B^k U(t-k) + \Xi(t)
\end{equation}
where $A^k$ is the $k$th ARX coefficient matrix, $B^k$ is the $k$th exogenous input coefficient matrix, and $\Xi(t)$ is the residual. If input force or ground motion is not available, the AR model can be applied instead of the ARX model. The second-order ordinary difference equation of motion of a perfectly elastic structure can be represented as a multivariate ARX model with the order of 2 for both $p$ and $q$ [11]. For shear structures, the diagonal elements of the $A^1$ matrix, denoted by $A^1_{ii}$, are functions of the story stiffness, $k_i$. The DSFs for this model are various functions of the coefficient matrices, such as all elements of the $A^1$, $B^q$, diagonal elements of the $A^q$, eigenvalues, and eigenvectors.

2.2. Damage identification
We first describe the conventional sequential change point detection method that processes the data sequentially considering the prior distribution of the time of damage and the pre- and post-damage distributions of the DSF known a priori. We then describe the method that addresses the cases when the post-damage distribution of the DSF is unknown.

2.2.1. Sequential change point detection method with known feature distributions
The single-fault sequential change point detection method addresses the problem of detecting a fault that happens at a random time $\lambda$ as quickly as possible under a constraint of the probability of falsely declaring a fault before it happens. The sequence of observations $\{X(t), t \geq 1\}$ is assumed to consist of independent and identically distributed random variables that have the distribution $f_0$ before the fault occurs (i.e., $t < \lambda$) and $f_1$ after the fault (i.e., $t \geq \lambda$). The distributions $f_0$ and $f_1$ are obtained prior to the analysis from simulations or experiments. The algorithm that addresses the problem of unknown distribution $f_1$ is explained in the next section. The optimal sequential procedure for solving the problem is to perform the following hypothesis test at each time $n$ based on the observations $F_n(X) = \{X(t), 1 \leq t \leq n\}$ [28]:
\begin{align}
H_0 & : \lambda > n, \\
H_1 & : \lambda \leq n.
\end{align}
This hypothesis test is equivalent to finding the fault detection time $\tau$ according to the threshold test on the posterior probability ratio at time $n$ for the false alarm rate $P(\tau < \lambda) \leq \alpha$:
\begin{equation}
\Lambda_n(X) = \frac{P(\lambda \leq n | F_n(X))}{P(\lambda > n | F_n(X))} \geq B_{\alpha},
\end{equation}
where the threshold $B_{\alpha}$ is defined as $(1 - \alpha)/\alpha$. This posterior probability ratio in Eq. (5) can be expanded as
\begin{equation}
\Lambda_n(X) = \frac{P(\lambda \leq n | F_n(X))}{P(\lambda > n | F_n(X))} = \sum_{k=-\infty}^{\infty} \sigma(k) \prod_{l=-\infty}^{n-1} f_0(X_l) \prod_{r=0}^{n} f_1(X_r) = \frac{\sum_{k=-\infty}^{\infty} \sigma(k) \prod_{r=0}^{n} f_1(X_r)}{\sum_{k=-\infty}^{\infty} \sigma(k)}.
\end{equation}
where $\sigma(n)$ is the prior distribution of $\lambda$ (i.e., $\sigma(n) = P(\lambda = n)$). The fault detection time $\tau$ is then defined as the earliest time when $\Lambda_n(X)$ exceeds the threshold $B_{\alpha}$:
\begin{equation}
\tau = \inf\{n : \Lambda_n(X) \geq B_{\alpha}\}.
\end{equation}
The Shiryaev–Roberts–Poljak (SRP) procedure [28] achieves the minimum asymptotic detection delay [12]
\begin{equation}
D_n^\alpha(\tau) = \mathbb{E}_n[(\lambda - \tau)^m | \tau \geq \lambda] \leq \left[ \frac{\log \alpha}{\mathbb{E}_n[X] + \alpha} \right]^m,
\end{equation}
where $\mathbb{E}_n$ denotes expectation with respect to the prior of $\lambda$, $m$ denotes asymptotic upper and lower bounds as $\alpha \to 0$, and typically $m = 1$ or 2. The delay bound is a function of the false alarm rate $\alpha$, the inverse of the mean of the prior for $\lambda$, denoted
by $d$, and the distance measure $q_1(X)$ between the two probability densities $f_0$ and $f_1$:

$$q_1(X) = \int f_1(x) \log \frac{f_1(x)}{f_0(x)} \mu(dx).$$

(9)

where $\mu(dx)$ is the Lebesgue measure. We can replace $\mu(dx)$ with $dx$ if we exclude discontinuous probability densities for $f_1(x)$ and $f_0(x)$. This bound is the minimum asymptotic delay achievable by any method with false alarm rate $\alpha$. Furthermore, if the test statistic in Eq. (5) can be computed for $n \geq 1$ using the recursion [12]

$$\Lambda_n(X) = \left( \frac{\Pi_{n-1}}{\Pi_n} \Lambda_{n-1} + \frac{\sigma(n)}{\Pi_n} f_1(X_n) \right) f_0(X_n)$$

(10)

$$\Lambda_0 = \frac{\sigma(0)}{1 - \sigma(0)},$$

(11)

where $\Pi_n$ is the prior complementary cumulative distribution of $\lambda$ (i.e., $\Pi_n = P(\lambda > n)$).

2.2.2. Sequential change point detection method with an unknown feature distribution for damage

In practice, the distribution $f_1$ is most likely to be unknown to us unless we have the means of exactly predicting how damage will be incurred to the structure. In such cases, $f_1$ can be estimated from the data, and then $\Lambda_n(X)$ in Eq. (6) can be computed accordingly. We present two methods to estimate $f_1$. First, we estimate by the maximum likelihood method such that the posterior probability ratio $\Lambda_n(X)$ is maximized. For example, let $f_0$ and $f_1$ be $\mathcal{N}(0, \sigma)$ and $\mathcal{N}(\mu, \sigma)$, respectively, where $\mathcal{N}(\mu, \sigma)$ is the Gaussian distribution with mean $\mu$ and standard deviation $\sigma$. These distributions can be substituted into the Eq. (6) as follows:

$$\Lambda_n(X) = \frac{\sum_{k=0}^{\lambda} \alpha(k) \prod_{r=1}^{n} e^{X_r - \mu} / \sigma^2}{\sum_{k=n+1}^{\lambda} \alpha(k)} = \frac{\sum_{k=0}^{\lambda} \alpha(k) \prod_{r=1}^{n} e^{X_r - \mu} / \sigma^2}{\sum_{k=n+1}^{\lambda} \alpha(k)}.$$  

(12)

By the Jensen’s inequality, Eq. (12) has the following lower bound:

$$\log \Lambda_n(X) \geq \frac{\sum_{k=0}^{\lambda} \alpha(k) \log \prod_{r=1}^{n} e^{X_r - \mu} / \sigma^2}{\sum_{k=0}^{\lambda} \alpha(k)} + \log \frac{\sum_{k=n+1}^{\lambda} \alpha(k)}{\sum_{k=n+1}^{\lambda} \alpha(k)} = \frac{\sum_{k=0}^{\lambda} \alpha(k) \left( \sum_{r=1}^{n} e^{X_r - \mu} / \sigma^2 \right)}{\sum_{k=0}^{\lambda} \alpha(k)} + \log \frac{\sum_{k=n+1}^{\lambda} \alpha(k)}{\sum_{k=n+1}^{\lambda} \alpha(k)}.$$  

(13)

Because of the complexity of the Eq. (12) when differentiated, we will instead find $f_1$ that maximizes the lower bound in Eq. (13). Let the lower bound in Eq. (13) be denoted as $\Lambda_n(X)$. The optimal estimation of the parameter $\mu$ of $f_1$, denoted as $\hat{\mu}$, can be found as follows:

$$\frac{d}{d\mu} \Lambda_n(X) = \frac{d}{d\mu} \left( \sum_{k=0}^{\lambda} \alpha(k) \left( 2\mu \sum_{r=1}^{n} X_r - \mu^2 (n-k+1) \right) \right) = \frac{2 \sum_{k=0}^{\lambda} \alpha(k) \sum_{r=1}^{n} X_r - \mu^2 (n-k+1)}{\sigma^2 \sum_{k=0}^{\lambda} \alpha(k)} = 0,$$

$$\hat{\mu} = \frac{\sum_{r=1}^{n} X_r}{n-k}.$$  

(14)

Another challenge for using the change point detection method with an unknown distribution of DSF after damage is that the recursive form in Eq. (10) is no longer equivalent to the original form in Eq. (6) computed using the most recent estimation of $\mu$ because the recursive form is computed using the entire history of the estimates of $\mu$ that is updated every time new data are collected. We impose a memory limitation to approximate the original form in Eq. (6) while keeping the computation simple. This approach limits the amount of accessible data for computing $\Lambda_n(X)$. In other words, only the most recent $m$ data, $X(t), n-m+1 \leq t \leq n$, are used for computing $\Lambda_n(X)$ as follows:

$$\Lambda_n(X) = \sum_{k=n-m+1}^{\lambda} \alpha(k) \prod_{r=1}^{n-m+1} e^{X_r - \mu} / \sigma^2 \sum_{k=0}^{\lambda} \alpha(k).$$  

(15)

The second method to estimate $f_1$ involves the Bayesian statistics. Using the Bayes rule and the law of total probability, the posterior distribution of $\lambda$ can be represented as

$$P(\lambda = k | \mathcal{F}_n(X)) = \int P(\mathcal{F}_n(X), \lambda = k) \frac{P(\lambda = k)}{P(\mathcal{F}_n(X))} = \int \frac{P(\mathcal{F}_n(X), \lambda = k, \mu) \mu P(\mu)}{P(\mathcal{F}_n(X))} d\mu \frac{P(\lambda = k)}{P(\mathcal{F}_n(X))}$$  

(16)

$$= \frac{\alpha(k) \prod_{r=1}^{n-m+1} f_0(X_r) \mu P(\mu) \mu P(\mu)}{P(\mathcal{F}_n(X))} = \frac{\alpha(k) \prod_{r=1}^{n-m+1} f_0(X_r) \mu P(\mu)}{P(\mathcal{F}_n(X))} = \frac{\alpha(k) \prod_{r=1}^{n-m+1} f_0(X_r) \mu P(\mu)}{P(\mathcal{F}_n(X))}.$$
If we use the conjugate prior distribution for $\mu$ with hyper-parameters $\mu'$ and $\sigma'$ for mean and standard deviation, respectively, Eq. (16) can be simplified as

$$P(\lambda = k|\mathcal{F}_n(X)) = \frac{\pi(k)\prod_{i=0}^{k-1} f_0(X_i) \prod_{i=k}^{n} f_1(X_i|\mu = 0)}{\mathcal{F}_n(X)} \frac{\sigma'}{\sigma''} \frac{1}{n^2 \sigma''^2} \frac{1}{2\pi} \phi_{-\lambda}(\mu_n)$$

where $\mu_n$ and $\sigma_n$ are the posterior mean and standard deviation of $\mu$ given the data $\{X(t), k \leq t \leq n\}$. Eq. (17) can be substituted into Eq. (5) to compute $A_n(X)$ as follows:

$$A_n(X) = \frac{\mathcal{F}_n(X)}{\mathcal{F}_n(X)} \frac{\sum_{k=0}^{n} \pi(k)\prod_{i=0}^{k-1} f_0(X_i) \prod_{i=k}^{n} f_1(X_i|\mu = 0) \phi_{-\lambda}(\mu_n)}{\sum_{k=0}^{\infty} \pi(k)\prod_{i=0}^{k-1} f_0(X_i) \prod_{i=k}^{n} f_1(X_i|\mu = 0)}$$

We can impose the memory limitation as before in Eq. (15) to reduce the computation load.

### 3. Validation

A set of experimental data is used to validate the damage diagnosis algorithm using the change point detection method, and numerous sets of synthetic data are used to further validate the consistency, robustness, and sensitivity of the algorithm. Because we want to declare the damage based on the data rather than relying on the prior information, an uninformative prior distribution $\pi(n)$ for the true damage occurrence time $\lambda$ is desirable. For mathematical convenience, the geometric distribution with a large mean (i.e., small $d$ in Eq. (8)) is selected as $\pi(n)$. It was found that the results are robust to the value of $d$ when $d \ll 1$. For this study, among various tested values of $d$, ranging from 0.0001 to 1, the ones smaller than 0.01 performed similarly well while the larger ones resulted in large false alarm rates.

#### 3.1. Experimental data

**3.1.1. Description of experiment**

The experiment involved a series of shake table tests of a 1:8 scale model for a four-story steel moment-resisting frame designed according to current seismic provisions (IBC-2003 [29], AISC-07-05 [30], FEMA-350 [31]). The experiment was conducted at the State University of New York at Buffalo [32,33]. Fig. 2 shows the structure after completion of the erection process on the shake table. The primary focus is on the model frame shown on the left. A total of 314 channels were used for the instrumentation of the test frame, including strain gauges, accelerometers, and displacement meters. For this analysis, we used the acceleration measurements at each floor including the ground motion and the roof response. The sampling rate was 128 Hz.

In order to introduce damage to the structure, the structure was subjected to a series of the scaled 1994 Northridge earthquake ground motions recorded at the Canoga Park, CA, station. The scaling factors were 40 percent, 100 percent, 150 percent, and 190 percent for a service level earthquake (SLE), a design level earthquake (DLE), a maximum considered earthquake (MCE), and a collapse level earthquake (CLE), respectively. Fig. 3 shows the story drift ratio (SDR) of the frame from elastic behavior (during the SLE) up to collapse for the first and the fourth story, respectively. The second and the third story had SDR similar to that of the first story. During the SLE the structure remained elastic (undamaged). During the DLE the structure reached a maximum SDR of about 1.6 percent with the inelastic action observed at the column base and first floor beams. Localized damage, such as local buckling of the flange plates, was not noticeable. During the MCE the frame reached a maximum SDR of about 5 percent with plastic deformation evident from local buckling of the plates that represented plastic hinge elements of the frame [33]. During the CLE test the frame experienced a maximum SDR of about 13 percent. The frame was also subjected to white noise excitations before and after each earthquake loading. After all the white noise responses are collected, the structure was subjected to another collapse level earthquake excitation, referred to as CLEF in Fig. 3, which resulted in a structural collapse with a full first three story collapse mechanism. For this analysis,
we used the white noise excitation responses collected before the SLE and DLE, and after the CLE prior to the structural collapse.

The acceleration response from each white noise excitation was segmented, and the AR coefficients were then extracted separately from each segment. To obtain the distributions of the DSF before and after the fault (i.e., $f_0$ and $f_1$, respectively), we assumed Gaussian distributions [9] and computed the sample means and variances of the DSF. The Gaussianity assumption was tested by performing normality tests on the DSF values under each type of damage. Quantile–quantile plots and the Pearson Chi-Squared test confirmed the normality assumption. It is also important to note that, even if the distribution of DSF failed this test, the proposed change point detection method would still function. The only issue is that it could experience longer delays than optimal. For $f_0$ we used the features extracted from the roof responses before the SLE and the DLE, and for $f_1$ we used the response after the CLE.

3.1.2. Results and discussion

Known post-damage feature distribution using a single-variate AR model. The results of the analysis using various combinations of the single-variate AR coefficients for the floor acceleration responses are presented in this section. Each acceleration measurement was segmented into 50 chunks, each of which includes 100 data points. Thus, each chunk took 100/128 s to be collected. Because the response signals are stationary, we can assume that each measurement chunk is obtained separately in different times. Fig. 4(a) shows the results of the Akaike Information Criteria (AIC) tests (see [26]). Based on the figure a model order of three, four or five can be selected. Typically the smallest order that can be used is preferred, and thus a model order of three is selected for this analysis. 150 values of the DSFs extracted from all three white noise excitations were fed into the algorithm sequentially to compute the test statistic $\Lambda_n$ for each floor response.

Fig. 4(b) shows the box plot of the first AR coefficients extracted from the roof acceleration responses. The center line and the edges of the boxes indicate the median and the 25th and the 75th percentiles of the coefficients, respectively. The whiskers of the boxes extend to the maximum and the minimum coefficients, excluding outliers, where the outliers for each box plot are defined as the data whose values are outside the $1.5 \times (\text{length of the box})$ below and above the box edges. Fig. 4(b) shows that the distribution of the first AR coefficient changes before and after damage in the structure. This change is reflected in the change in the test statistic value, as shown in Fig. 4(c); this test statistic is computed from the first AR coefficients for each floor acceleration responses. The thin dash line in Fig. 4(c) indicates the threshold value for the false alarm rate of $10^{-6}$ percent. The crossings between the dash line and the test statistic plots indicate the corresponding fault detection times. The detection delay is the difference between the fault detection time and the true fault time, which is 101. While the algorithm identified damage without a false alarm using the acceleration responses from the third and fourth floors and the roof, it triggered the false alarm (i.e., fault detection time $\leq$ true fault time) using the second floor data. Note that the detection delays are described in time steps, each of which corresponds to 100/128 s as explained above. The detection delays are very small, in the order of a few seconds, because the delays do not include the time interval between measurements. For example, if the measurements are collected every hour, then the test score for the algorithm is updated hourly. If the algorithm detects damage after say 5 sets of measurements after damage, the corresponding delay is between 4 and 5 h. However, the current model does not consider the time between data collections (and thus also disregard the duration of strong motion or any incidence that causes damage) and instead include only the duration of measurements, which is very short.
The performance of the change point detection algorithm is compared with that of the SPRT introduced in Section 1. For SPRT, the Gaussian distribution is assumed for the data, and the null hypothesis that the mean of the data is same as the mean of the undamaged case (i.e., no damage) is tested against the alternate hypothesis that the mean is different (i.e., damage). See Gosh [19] and Sohn et al. [6] for more detailed description of the procedure. Fig. 5(a) shows the test statistic $Z_n$ for the SPRT, which is the logarithm of the likelihood ratio at each time step $n$. The likelihood ratio is defined as the ratio of the conditional probability of observing the data given that the alternative hypothesis is true to that the null hypothesis is true. Given the upper constraints on false positive and false negative errors, $\alpha$ and $\beta$, respectively, there are three possible outcomes of the SPRT: (1) the null hypothesis is accepted if $Z_n \leq \ln \beta = (1/\alpha)\ln \alpha$; (2) the alternative hypothesis is accepted if $Z_n \geq \ln \beta/(1-\beta)$; and (3) no decision is made otherwise (i.e., $\ln \beta/(1-\beta) \leq Z_n \leq \ln \beta/(1-\alpha)$). Note that because the purpose of the SPRT is to determine the state of the data, which corresponds to either one of the two hypotheses, with a minimum number of data, no decision means that the algorithm does not have enough information to make any decision about the data. Thus, it waits for the next sample to arrive. As explained before, the test statistics value decreases initially when the structure is undamaged and then starts increasing when the damage occurs at time step 101. This occurs because the SPRT assumes that all the data are from the same distribution with no consideration of change point in time. If we set $\alpha$ and $\beta$ to be $10^{-6}$ percent, same as the results in Fig. 4(c), the threshold value is $-18.42$ for accepting the null hypothesis and $18.42$ for the alternative; thus this method will accept that the structure is intact after observing the first 9 data. Ignoring the lower bound and considering only the upper bound, in other words, only waiting for damage assertion, results in no damage detection before the time step of 150. Even if the increasing rate of $Z_n$ is fast after the damage occurs at time 101, it takes some time to recover the initial drop. To address this problem, the method is modified to eliminate the data that are labeled to be from the undamaged structure when $Z_n$ is computed. Specifically, the test statistic $Z_n$ is initialized to zero whenever the null hypothesis is accepted. In other words, $Z_n$ is set back to zero when its value is below $-18.42$. The resulting $Z$ is shown in Fig. 5(b) along with the logarithm of the test statistic $\Lambda_n$. The performance is much better with the modification that $Z_n$ value starts increasing soon after the damage is introduced, but the rate of increase is slower than that of the change point detection method.

Although the performance of the algorithm based on the single-variate AR is excellent for some acceleration responses within an allowable false alarm rate set by the user, the single-variate AR model did not make it clear which sensor data we should rely on for damage identification. Moreover, although the first three stories are more severely damaged than the fourth story, as shown in Fig. 3, the rate of the increase in the $\Lambda_n$ statistic for each floor after the true fault time does not correspond to the severity of damage for each floor. Therefore, the single-variate AR coefficients cannot localize damage on the basis of sensor locations. As a result, the features based on multivariate models may provide more reliable damage detection because they are independent of the sensor selection.
Where we used all the elements of the experiments or simulations is necessary for the bounds to be valid. In addition, because the delay and the corresponding false alarm rate are estimated by averaging, a sufficiently large number of limit of the false alarm rate going to zero, the resulting detection delay may be different from the asymptotic bound. In Fig. 4(d), similar results are obtained using the DSFs defined as combinations of the first three AR coefficients, and their fault detection times using the roof responses are summarized in the first three rows of Table 1 for various false alarm rates. Different DSFs listed in Table 1 are selected on the basis of the suggestions by Nair et al. [9] and Cheung and Kiremidjian [11] to be particularly efficient for damage diagnosis. Although the use of only the first AR coefficient as the DSF performs the best, the performance differences are insignificant. Note that some empirical values are lower than the asymptotic bounds, indicated by a dash line in Fig. 4(d). Because the asymptotic bounds hold true in the ground motion. Fig. 6 shows the four diagonal elements of the acceleration responses are collected from four different locations in the structure and the input force is applied by change much. These behaviors are reflected in Fig. 7(a), where the diagonal elements of $A_1$ cannot localize the damage.

Various other DSFs are tested, and Fig. 7(b)–(e) shows the $\Lambda_n$ statistic results that use as a DSF four diagonal elements of $A^1$, four eigenvalues of $A^1$, all elements of $A^1$, and the nonzero element of $B^1$, respectively. Note that the DSFs for four diagonal elements of $A^1$, four eigenvalues of $A^1$, and all elements of $A^1$ are multidimension. The second part of Table 1 summarizes the fault detection times for various DSFs based on the multivariate AR model at various false alarm rates. The table shows that individual eigenvalue of the $A^1$ matrix is more sensitive to damage than its individual diagonal element. In addition, using a multidimensional DSF usually results in a more accurate fault detection time. The results also show that the algorithm tends to perform better when the false alarm rate is lower. Among all the features, the nonzero element of $B^1$ (its first element) has the most reliable performance because its $\Lambda_n$ statistic does not fluctuate much when the structure is undamaged.

From the definition of $B_n$, as the false alarm rate decreases, the threshold value increases, and, as a result, the detection delay increases (see Fig. 4(d)). Similar results are obtained using the DSFs defined as combinations of the first three AR coefficients, and their fault detection times using the roof responses are summarized in the first three rows of Table 1 for various false alarm rates. Different DSFs listed in Table 1 are selected on the basis of the suggestions by Nair et al. [9] and Cheung and Kiremidjian [11] to be particularly efficient for damage diagnosis. Although the use of only the first AR coefficient as the DSF performs the best, the performance differences are insignificant. Note that some empirical values are lower than the asymptotic bounds, indicated by a dash line in Fig. 4(d). Because the asymptotic bounds hold true in the limit of the false alarm rate going to zero, the resulting detection delay may be different from the asymptotic bound. In addition, because the delay and the corresponding false alarm rate are estimated by averaging, a sufficiently large number of experiments or simulations is necessary for the bounds to be valid.

**Table 1**

<table>
<thead>
<tr>
<th>DSF</th>
<th>1%</th>
<th>$10^{-3}$%</th>
<th>$10^{-4}$%</th>
<th>$10^{-5}$%</th>
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<td>102</td>
<td>103</td>
</tr>
<tr>
<td>${a_1, a_2}$</td>
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<td>103</td>
<td>104</td>
<td>105</td>
</tr>
<tr>
<td>${a_1, a_2, a_3}$</td>
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<td>102</td>
<td>103</td>
<td>104</td>
</tr>
<tr>
<td>$[A_{1,1}]_k$</td>
<td>14</td>
<td>35</td>
<td>41</td>
<td>44</td>
</tr>
<tr>
<td>$[A_{1,1}, A_{1,2}, A_{1,3}, A_{1,4}]_k$</td>
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<td>104</td>
<td>105</td>
<td>107</td>
</tr>
<tr>
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<td>42</td>
<td>107</td>
<td>109</td>
</tr>
<tr>
<td>$[\text{eig}(A^1), \text{eig}(A^1), \text{eig}(A^1), \text{eig}(A^1)]_k$</td>
<td>29</td>
<td>30</td>
<td>104</td>
<td>105</td>
</tr>
<tr>
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<td>102</td>
<td>103</td>
<td>104</td>
</tr>
<tr>
<td>$[B_{1,1}]_k$</td>
<td>103</td>
<td>104</td>
<td>105</td>
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</tbody>
</table>

The analysis results are presented for the cases where we used only the elements of the $A^k$ coefficient matrices separately. $A^k$ is a 4 × 4 matrix and $B^k$ is a 4 × 1 vector because the acceleration responses are collected from four different locations in the structure and the input force is applied by the ground motion. Fig. 6 shows the four diagonal elements of the $A^1$ matrix, and Fig. 7(a) shows the test statistic for each diagonal element. Although the distributions of $A_{1,1}^1$ and $A_{1,4}^1$ change significantly after damage, $A_{1,2}^1$ and $A_{1,3}^1$ do not change much. These behaviors are reflected in Fig. 7(a), where the $\Lambda_n$ statistics for $A_{1,1}^1$ and $A_{1,4}^1$ show significant changes near the true fault time, while the $\Lambda_n$ statistics for $A_{1,2}^1$ and $A_{1,3}^1$ do not. Given that the first three stories are more severely damaged than the fourth story, the diagonal elements of $A^1$ cannot localize the damage.

**Known post-damage feature distribution using a multivariate ARX model:** The analysis results are presented for the cases where we used all the elements of the $A^k$ coefficient matrices, only their diagonal elements, their eigenvalues, and the nonzero elements of the $B^k$ coefficient matrices separately. $A^k$ is a 4 × 4 matrix and $B^k$ is a 4 × 1 vector because the acceleration responses are collected from four different locations in the structure and the input force is applied by the ground motion. Fig. 6 shows the four diagonal elements of the $A^1$ matrix, and Fig. 7(a) shows the test statistic for each diagonal element. Although the distributions of $A_{1,1}^1$ and $A_{1,4}^1$ change significantly after damage, $A_{1,2}^1$ and $A_{1,3}^1$ do not change much. These behaviors are reflected in Fig. 7(a), where the $\Lambda_n$ statistics for $A_{1,1}^1$ and $A_{1,4}^1$ show significant changes near the true fault time, while the $\Lambda_n$ statistics for $A_{1,2}^1$ and $A_{1,3}^1$ do not. Given that the first three stories are more severely damaged than the fourth story, the diagonal elements of $A^1$ cannot localize the damage.

Various other DSFs are tested, and Fig. 7(b)–(e) shows the $\Lambda_n$ statistic results that use as a DSF four diagonal elements of $A^1$, four eigenvalues of $A^1$, all elements of $A^1$, and the nonzero element of $B^1$, respectively. Note that the DSFs for four diagonal elements of $A^1$, four eigenvalues of $A^1$, and all elements of $A^1$ are multidimension. The second part of Table 1 summarizes the fault detection times for various DSFs based on the multivariate AR model at various false alarm rates. The table shows that individual eigenvalue of the $A^1$ matrix is more sensitive to damage than its individual diagonal element. In addition, using a multidimensional DSF usually results in a more accurate fault detection time. The results also show that the algorithm tends to perform better when the false alarm rate is lower. Among all the features, the nonzero element of $B^1$ (its first element) has the most reliable performance because its $\Lambda_n$ statistic does not fluctuate much when the structure is undamaged.
Fig. 6. Box plot of the diagonal elements of $A^1$: (a) $A^1_{1,1}$; (b) $A^1_{2,2}$; (c) $A^1_{3,3}$; (d) $A^1_{4,4}$.

Fig. 7. $\Lambda_n$ statistics: (a) each diagonal element of $A^1$; (b) four diagonal elements of $A^1$; (c) four eigenvalues of $A^1$; (d) all elements of $A^1$; (e) first element of $B^1$. 
Unknown post-damage feature distribution using a single-variate AR model: The results of the analysis using the first AR coefficient of the roof acceleration responses are presented in this section. Fig. 8 shows the DSF values from the first single-variate AR coefficients extracted from the white noise response of the frame and their quantile–quantile plot against the standard Gaussian distribution before damage happens. We can observe that the DSF values closely follow the Gaussian distribution before damage and the damage happened around time 79 s. Thus, the Gaussian assumption for the distribution of the DSF is valid. Fig. 9 shows the estimated \( \mu \) value, as a function of time, estimated using the maximum likelihood method. The estimate \( \mu \) quickly converges to the mean DSF value for the undamaged structure, \( -2.12 \), in the first 25 time steps. It then gradually approaches the true mean for the damaged case, \( -2.54 \), after the true damage time of 101. In this analysis, only the mean of \( f_1 \) is considered unknown, and the standard deviation is assumed to be the same as that of \( f_0 \). Although their true standard deviations, 0.13 and 0.087, are not identical, the algorithm was able to identify damage as shown below.

The detection delays computed using the maximum likelihood method and the Bayesian method are presented in Figs. 10 and 11, respectively. Similar to the simulation results, the detection delay increases as we set the allowable false alarm rate \( \alpha \) small. Also, a shorter memory leads to a longer detection delay for the same reason explained in Section 3.2.2. Knowing the true mean case performed nearly as good as the asymptotic bound, and having the full memory was about 7 time steps slower than knowing the true mean case for the maximum likelihood method and about 1 time step for the Bayesian method. The memory length of 20 resulted in the equivalent detection delay to the full memory for the maximum likelihood method, and the memory length of 10 for the Bayesian method. Overall, the Bayesian method performed much better in terms of minimizing the detection delay. These results are comparable to the simulation results since the difference between the means of \( f_0 \) and \( f_1 \) normalized by their known or assumed standard deviation \( \sigma \) is 3.23, which is similar to 3 for the simulation.

3.2. Simulated data

3.2.1. Description of simulation

In order to test the algorithm with multiple sets of data with varying statistical properties, large sets of simulated data are generated. The data consist of random numbers with the Gaussian distribution. The simulation involved 10,000 sets of 600 independent random numbers whose mean changes from 0 to \( \mu \) after the 100th random number. Thus, first 100
numbers are distributed around 0, and the next 500 numbers are around $\mu$. The simulation was repeated for various values of $\mu$ including 0.5, 1, 2, 3, and 5. The variance of the random number remained constant as 1. Using these data sets, the consistency and the robustness of the algorithm were assessed by investigating their false alarm rates and detection delays. For each of the data sets, we applied the change point detection algorithm for known post-damage feature distribution cases and unknown cases using the maximum likelihood method and the Bayesian method with various degree of memory limitation. The false alarm rate and the detection delay results for unknown post-damage feature distribution cases are compared with the theoretical asymptotic bounds and the case where we know the true $f_1$. In the unknown distribution cases, a Gaussian distribution with mean of 3 and standard deviation of 3 is used as the diffused prior distribution for $\mu$.

3.2.2. Results and discussion

The distribution of the detection time $\nu$ when $\mu$ is 3 is shown in Fig. 12(a), and that of the estimated $\mu$ using the maximum likelihood method is shown in Fig. 12(b). We can observe that the detection delay is distributed symmetrically, and both the detection delay and $\mu$ do not have significant outliers nor multiple modes. Thus, the algorithm performed consistently for each simulation. Moreover, the estimated $\mu$ starts approaching the true $\mu$ as soon as the data shifts the mean at $n=101$. Note that the estimated $\mu$ has the mean near zero for $n \leq 100$, and its variance decreases as more data is collected. Figs. 13 and 14 show the detection delay and the false alarm rate for $\mu$ of 3 computed from the maximum likelihood method and the Bayesian method with various memory limitations, respectively. As expected, the algorithm detects the change faster when we allow a larger false alarm rate $\alpha$. Having a shorter memory helps to reduce false alarm but results in longer detection delay. The detection delay increases faster as $\alpha$ decreases (i.e., the threshold increases) with a shorter memory. Therefore, a shorter memory makes the algorithm less sensitive to the changes in the data and respond slower. The memory length of 20 data performs almost equivalently to having the full memory in terms of detection delay. This is because having a shorter memory corresponds to making a rougher approximation in exchange of computational load. Knowing the true $\mu$ improves the detection delay by a few unit times and approaches the theoretical asymptotic lower bound closely. The Bayesian method resulted in the detection delays much closer to knowing the true mean case, which are at most 18 percent longer with full memory, than the maximum likelihood method, which are at most about 250 percent longer with full memory. The maximum likelihood method, however, provided an insight into how the post-damage distribution is estimated, by directly showing that the estimated parameter of the unknown distribution converges to the true value quickly after the change.

![Fig. 10. Detection delays using the maximum likelihood method for the experimental data.](image1)

![Fig. 11. Detection delays using the Bayesian method for the experimental data.](image2)
point, as shown in Fig. 12(b). Similar results were obtained for other values of \( \mu \), and the results are summarized in Fig. 15 using the Bayesian method. The figure shows the detection delay for various values of \( \mu \). The false alarm rate was fixed at 1 percent. As the value of \( \mu \) decreases, the two distributions for pre- and post-damage cases become close to each other, which in turn worsens the damage detection delay for the same false alarm rate.

4. Conclusions

We have developed a damage diagnosis algorithm based on time-series models and a single fault sequential change point detection method that addresses the cases when the feature distribution is unknown when the structure is damaged. The case of unknown distribution of the DSF is of great interest because in practice we are not likely to have this
information. Following the collection of structural responses, the algorithm extracts a damage sensitive feature (DSF) using the autoregressive model and classifies the DSF into a damage state using a sequential change point detection method. The classification using the change point detection method involves sequential hypothesis test of the DSF extracted at each time step. The sequential nature of this method allows us to make a decision using a series of observations sequentially and eliminates the need to wait for a set of features to be collected for statistical analysis. Therefore, it is particularly suitable for a long-term periodic monitoring of a structure. The sequential test uses the DSF’s pre-damage distribution and the estimated post-damage distribution. The post-damage distribution of the DSF is updated every time a new set of data is available. This updating scheme is developed using the maximum likelihood estimation and the Bayesian approach. In addition, an approximate method with a memory limitation is applied to increase the computational efficiency of the algorithm.

For validation, we applied the algorithm to a set of white noise shake table test data collected from a four-story steel moment-resisting frame and multiple sets of simulated data. The results show that the algorithm can reliably detect damage within the false alarm rate set by the user and estimate the distribution of the DSF for damaged case using the maximum likelihood method and the Bayesian method. The experimental results showed that the detection was around 1 and 7 time steps slower than the asymptotic bound using the maximum likelihood and Bayesian methods, respectively, with a memory length of 10 data points. The simulation results showed that the algorithm can sequentially identify damage with the fault detection time delay close to the theoretical lower asymptotic bound. The Bayesian method provided fault detection times that are at most 18 percent longer than the asymptotic bound with a memory length of 10 data points. In addition, the empirical false alarm rate was similar to the allowable false alarm rate afforded even with the approximate method with the memory limitation. The overall performance of the Bayesian method was better than that of the maximum likelihood method for both the simulation and the experimental data, but the maximum likelihood method provided a heuristic approach to the problem by directly showing how the estimated parameter of the unknown distribution behaves.

To further improve this algorithm, it can be applied to other types of DSFs to analyze nonstationary responses. Moreover, we are also investigating the detection of multiple damage states at multiple damage locations simultaneously using t
he change point detection method. Last, we need to apply the algorithm to more experimental and field data for further validation.

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